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SHAPE OF AN INCOMPRESSIBLE, WEAKLY CONDUCTING JET IN A STRONG ELECTRIC FIELD

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The question of jet formation in an electric field has mainly been investigated experimentally. The generation of jets of various fluids in an electrostatic field was systematically studied by Zeleny [1], who found a number of empirical conditions for the interelectrode voltage and the fluid head governing the transition from droplets to a continuous jet flow. In [2] the effect of the field strength and the flow rate on the length of the continuous part of the jet and in [3] the effect of conductivity on the breakup length were investigated for fluids with a broad range of viscosities, conductivities and surface tensions (water−glycerol mixtures, salt solutions, organic and inorganic oils, etc.). Many studies have been devoted to the atomization of charged jet flows. For example, in [4] jets of insulating cryogenic liquids were examined in connection with the possibility of obtaining spherical microtargets of controllable size for laser thermonuclear synthesis. In [5, 6] electrostatic microfiber spinning from polymer solutions and melts was studied experimentally.

It is assumed that the fluids being investigated possess ionic conductivity with a characteristic electric time parameter less than the characteristic capillary outflow time. Because of the small relaxation time in the immediate vicinity of the fluid-emitting electrode the charge flows away towards the surface of the jet. The electric forces are determined by the interaction of the external field and the injected surface charges, the mutual repulsion of the latter, and the polarization interactions. For a thin, weakly decaying jet profile the polarization forces are small, there are no electric body forces, and the electric and hydrodynamic fields interact across the jet boundary. Strong electrostatic fields are considered. For these fields the field strength associated with charge transfer UI exceeds the initial power of the hydrodynamic flow $\rho Q^3/2\pi^2 r_0^4$, i.e., the parameter $\delta = \rho Q^3/2\pi IUr_0^4 \ll 1$ (I is the current transported by the jet, U is the potential difference on the coordinate interval investigated, ρ and Q are the density and volume flow rate of the fluid, and r_0 is the initial radius of the jet). This makes it possible to omit the initial conditions of jet

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generation at the fluid-injecting electrode. In [7] the role of surface tension, the mutual repulsion of the surface charges, and the interaction of polarization charges were analyzed. These forces make a small contribution to the velocity and the dependence of the radius on the longitudinal coordinate. We shall not be concerned with them here.

1. Formulation of the Problem. We will consider the linear motion of a net in the cylindrical coordinate system r, z. The z axis coincides with the axis of symmetry of the circular jet, and the plane z = 0 passes through the end section of the capillary. The solution of the hydrodynamic equations for the velocity V and the pressure p is found in the form:

$$V_z = \sum_0 a_n(z) r^n, \quad p = \sum_0 p_n(z) r^n, \tag{1.1}$$

where the coefficients a_n and p_n are functions only of z. The quantities with the dimension of length are normalized on r_0, and $Q/\pi r_0^2$ and $\rho Q^2/\pi^2 r_0^4$ are taken as the velocity and pressure scales, respectively. Using the relation between V_r and V_z obtained from the continuity equation, for the radial velocity we obtain

$$V_r = -\sum_0 a_n'(z) \frac{r^{n+1}}{n+2}$$

(the prime denotes differentiation with respect to z). Substituting the velocities and pressure in the steady-state Navier–Stokes equations, we find that there are no odd powers in expansions (1.1). In what follows in series (1.1) we shall consider terms only up to the fourth order in r:

$$V_z = a_0 + a_2 r^2 + a_4 r^4, \quad V_r = -r\left(\frac{1}{2} a_0' + \frac{1}{4} a_2' r^2 + \frac{1}{6} a_4' r^4\right),$$
$$p = p_0 + p_2 r^2 + p_4 r^4. \tag{1.2}$$

Equating terms with like powers of r in the Navier–Stokes equations, we obtain

$$r^0\colon\ a_0 a_0' = -p_0' + \frac{1}{\mathrm{Re}}\left(a_0'' + 4a_2\right); \tag{1.3}$$

$$r^1\colon\ \frac{1}{4} a_0'^2 - \frac{1}{2} a_0 a_0'' = -2p_2 - \frac{1}{\mathrm{Re}}\left(2a_2' + \frac{1}{2} a_0'''\right); \tag{1.4}$$

$$r^2\colon\ a_0 a_2' = -p_2' + \frac{1}{\mathrm{Re}}\left(a_2'' + 16a_4\right). \tag{1.5}$$

Here, $\mathrm{Re} = \rho Q/\pi r_0 \mu$; μ is the viscosity of the fluid. In accordance with the chosen velocity expansion for the stream function we have $\psi = \Sigma a_n(z) r^{n+2}/(n+2)$. Confining our attention to terms of up to fourth order in r, from the condition of constancy of ψ on the surface of the jet we find the relation

$$a_0 f^2 = C - (1/2) a_2 f^4, \tag{1.6}$$

where r = f(z) is the shape of the jet in cylindrical coordinates, and the constant C = 1 is determined from the condition of conservation of the flow rate. For the boundary streamline from (1.6) we have

$$V_z f^2(z) = 1 + (1/2) a_2 f^4(z). \tag{1.7}$$

The infinite system of differential equations, of which relations (1.3)-(1.5) form an initial fragment, is such that in any number of equations taken in succession in order of increasing powers of r the number of unknowns will exceed by two the number of equations. By adding to (1.3)-(1.5) the conditions for the normal and tangential tensions on the boundary streamline and Eq. (1.6) we obtain a system closed with respect to the number of variables.

2. Jet in a Uniform Electric Field. The charge is injected into the jet at the fluid-emitting electrode. As a result of the short relaxation time $\tau = \varepsilon\varepsilon_0/\lambda$ (ε is the dielectric constant, and λ the conductivity) charge transfer takes place along the boundary streamline. Neglecting the ohmic current, for the electric surface current we write

$$I = \lim_{\Delta t \to 0} \frac{\sigma 2\pi f \Delta l}{\Delta t} = \sigma 2\pi f V_\tau.$$

Here we have taken into account only the contribution of convective charge transfer, $V_\tau = V_z\sqrt{1 + f'^2}$ is the velocity of the boundary streamline, and $\Delta\ell$ is the arc length of the lateral surface between nearby cross sections of the jet. Omitting the terms containing f'^2 and f^4, whose contribution to the solution is estimated below, and using (1.7), for the surface charge density we have

$$\sigma = If(z)/2Q. \tag{2.1}$$

Provided that the distortions of the external uniform field of strength E are small, the tangential component of the electric tension tensor is equal to $T_\tau = \sigma E$ [8]. The nondimensionalized equation of the tangential tensions on the jet boundary can be written in the form:

$$2a_2 - \frac{1}{2}a_0'' - \frac{1}{4}a_2''f^2 = \frac{1}{4}\frac{\mathrm{Re}}{s} \tag{2.2}$$

($s = \rho Q^3/2\pi^2 IEr_0^5$). The normal electric forces are small, so that for the normal tensions we obtain

$$-p_0 = p_2 f^2 + \frac{1}{\mathrm{Re}}\left(a_0' + \frac{3}{2}a_2' f^2\right). \tag{2.3}$$

In the six relations (1.3)-(1.6), (2.2), (2.3) there are six variables a_0, a_2, a_4, p_0, p_4, and f subject to determination. The solution is found in the domain in which for the unknown dependence of the jet radius f(z) we have $f'(z) \ll 1$. Assuming that $a_{2n} \gg a_{2n+2}$ and $a_2(z) = \text{const} + o(z)$, i.e., that the expansion of a_2 begins with a constant, we can drop the term $a_2''f^2$ from (2.2). Integrating (1.3) using (2.2), we find

$$\frac{a_0^2}{2} = -p_0 + \frac{1}{2s}z + \frac{2a_0'}{\mathrm{Re}} + \text{const}. \tag{2.4}$$

The second term in the pressure is obtained from (1.4) and (2.2)

$$p_2 = -\left(\frac{1}{8}a_0'^2 - \frac{1}{4}a_0 a_0''\right) - \frac{a_0'''}{2\,\mathrm{Re}}. \tag{2.5}$$

Substituting (2.1), (2.3), and (2.5) in (2.4) gives the following equation for a_0:

$$\frac{a_0^2}{2} = \frac{z}{2s} + \text{const} + \frac{3a_0'}{\mathrm{Re}} - \left(\frac{1}{8}a_0'^2 - \frac{1}{4}a_0 a_0''\right)f^2 - \frac{a_0''' f^2}{8\,\mathrm{Re}}.$$

Making use of the smallness of a_0' as compared with a_0, we write

$$a_0 = \sqrt{z/s + \text{const}}. \tag{2.6}$$

In form this expression is identical to the formula for the velocity of a jet in a uniform gravity field, but here the role of the acceleration of gravity is played by the quantity IE/ρQ. The corrections to the velocities and pressures, which are of high order in z, can be calculated using (2.6).

Considering that on the interval of z in question $a_0^2 \gg a_2$, from (1.6) we derive the dependence:

$$f^2(z) = \frac{1}{a_0} - \frac{1}{2}\frac{a_2}{a_0^3}.$$

The constant in (2.6) can be found from matching the solution obtained and the solution at the capillary. However, in the strong field approximation it can be omitted. The shape and the coefficients of the expansion of the velocities in dimensional form can be determined from the expressions

$$f(z) = (\rho Q^3/2\pi^2 IEz)^{1/4},\ a_0 = \sqrt{2IEz/\rho Q},\ a_2 = IE/4\mu Q, \tag{2.7}$$

and the pressure is found from (2.3), (2.5). As was to be expected, the shape and velocity in the force field do not contain the dependence on the initial velocity and radius of the jet.

Using (2.7), we will estimate the terms omitted in the various stages of the calculations. In obtaining (2.1) we neglected the terms f'^2 and $(1/2)a_2f^4$ in expression (1.7), which are equal to $f^2/16z^2$ and $\mathrm{Re}\,r_0/16z$, respectively. The ratios of the terms in the velocity V_z have the values: $a_4f^4/a_0 = \frac{3}{256}\mathrm{Re}\frac{r_0 f^2}{z^3} + \frac{495}{1024}\frac{f^4}{z^4}$, $a_2f^2/a_0 = \frac{\mathrm{Re}}{8}\frac{r_0}{z}$. Choosing the greatest, we obtain the conditions for the region of existence of the solution found:

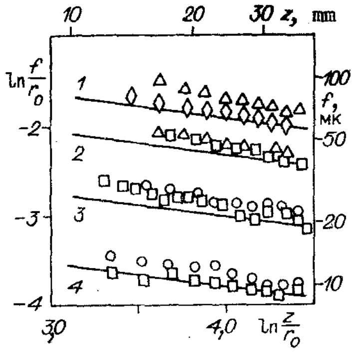

Fig. 1

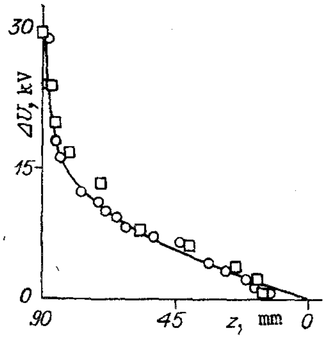

Fig. 2

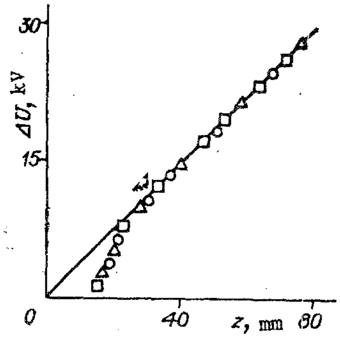

Fig. 3

$$(\mathrm{Re}/8)(r_0/z) \ll 1; \tag{2.8}$$

$$f/z \ll 1. \tag{2.9}$$

The first limitation is imposed on the viscosity of the fluid, the second on the field.

The viscous velocity field is important in this problem as the medium converting the surface electric forces into volume kinetic energy. Despite the fact that the solution obtained does not exist when $\mu = 0$, there is an interval of z determined by the condition (2.8) on which the viscosity corrections to the velocity and the shape can be neglected. In this case the energy of the electric field is converted into kinetic energy almost without dissipation in the viscous tension field.

The shape of the jet in a uniform electric field was investigated experimentally in accordance with the scheme described in [7]. The gap between the electrodes of the plane-parallel capacitor was 40 mm, the radius of the capillary $r_0 = 0.44$ mm. In Fig. 1 we have plotted the experimental points for the dependence of the radius of the jet on z. The following notations has been used for the fluids: circle) glycerol (GL), $\rho = 1.26$ g/cm³, $\mu = 13$ p; square) cyclohexanol (CHL), $\rho = 0.96$, $\mu = 0.55$; triangle) dimethylphthalate (DMP), $\rho = 1.19$, $\mu = 0.16$; diamond–dibutylphthalate (DBP), $\rho = 1.04$, $\mu = 0.21$. The straight lines correspond to (2.7): 1-4) s = 0.4, 0.27, 0. 135, 0.062. The jets had the parameters: line 1) DBP, $Q = 6 \cdot 10^{-2}$ cm³/sec, $E = 7.5 \cdot 10^5$ V/m, $I = 0.335 \cdot 10^{-7}$ A; DMP, $Q = 10.8 \cdot 10^{-2}$, $E = 7.5 \cdot 10^5$, $I = 2.32 \cdot 10^{-7}$; 2) DMP, $Q = 5.55 \cdot 10^{-2}$, $E = 7.5 \cdot 10^5$, $I = 1.57 \cdot 10^{-7}$; CHL, $Q = 4.93 \cdot 10^{-2}$, $E = 5.75 \cdot 10^5$, $I = 1.12 \cdot 10^{-7}$; 3) CHL, $Q = 2.5 \cdot 10^{-2}$, $E = 9.5 \cdot 10^5$, $I = 1.42 \cdot 10^{-7}$; GL, $Q = 1.62 \cdot 10^{-2}$, $E = 5.88 \cdot 10^5$, $I = 0.865 \cdot 10^{-7}$; 4) CHL, $Q = 6.4 \cdot 10^{-3}$, $E = 7.5 \cdot 10^5$, $I = 0.69 \cdot 10^{-7}$; GL, $Q = 5.88 \cdot 10^{-3}$, $E = 7.43 \cdot 10^5$, $I = 0.69 \cdot 10^{-7}$.

In the experiments Re/8 varied from $5 \cdot 10^{-4}$ to 0.7. In the inequality $z \gg s^{4/5} r_0$, equivalent to condition (2.9), the parameter $s^{4/5}$ did not exceed 0.5. Hence, in accordance with (2.8), (2.9), when $z \gg 0.7 r_0$ in these experiments we must expect the shape of the jet to approach (2.7). In reality this takes place when $z \sim (20\text{-}30) r_0$ because of the fact that there is a fairly extensive zone near the capillary in which the jet drawing mechanism differs from that considered. It is also clear that dependence (2.7) is approached more rapidly by jets with smaller Re, in accordance with (2.8).

3. Jet in a Nonuniform Electric Field. Condition (2.8), satisfaction of which means that the viscosity does not contribute to the dominant terms of the velocities and the dependence of the jet radius on the longitudinal coordinate, makes it possible to integrate the equations of motion in an arbitrary electrostatic field. Assuming, as before, that the acceleration of the jet is determined by the interaction of the electric field and the surface charge tangential to the jet boundary, we write the equations of the quasi-one-dimensional approximation in the form [7]:

$$\frac{d}{dz} \rho Q V_z - \frac{IE}{V_z} = \rho g S; \tag{3.1}$$

$$V_z S = Q; \tag{3.2}$$

$$V_r = -\frac{r}{2} \frac{dV_z}{dz}. \tag{3.3}$$

Here, the longitudinal velocity V_z is a function only of z; S is the area of the jet cross section. The relation (3.2) is the continuity equation averaged over the cross section of the jet, and Eq. (3.3) serves for determining the radial velocity. Integrating (3.1)-(3.3) and using the relation $E = -\nabla U$, we obtain $\rho Q\left(V_z^2(z) - V_z^2(z_0)\right)/2 = I\left(U(z_0) - U(z)\right) + g(z - z_0)\rho Q$ (z_0 is the potential reference point). The individual terms of the relation obtained are the strengths of the hydrodynamic flow, the electric field, and the gravity field. For the potential we have

$$U(z) = U(z_0) + \frac{\rho Q^3}{2\pi^2 I r^4(z_0)}\left(1 - \frac{r^4(z_0)}{r^4(z)}\right) + \frac{\rho g Q}{I}(z - z_0). \tag{3.4}$$

In accordance with (3.4), we experimentally determined the potential distribution on the axis of a plane-hyperboloid of revolution electrode system using glycerol jets. The glycerol was injected through an opening in the center of the horizontal flat upper metal electrode mounted at a distance b = 91 mm from the vertex of a hyperboloid with focal length c = 91.425 mm. The equation of the hyperboloid was $z^2/b^2 = 1 + r^2/(c^2 - b^2)$, the z coordinate was reckoned from the plane electrode, and $z_0 = 0$. To the upper electrode we supplied a voltage U_0 = 30 kV, and the hyperboloid was grounded. The theoretical potential distribution on the axis of this electrode system is given by the equation [9]

$$U(z) = U_0 - U_0 \ln\frac{c+z}{c-z}\Big/\ln\frac{c+b}{c-b}. \tag{3.5}$$

In Fig. 2 we have plotted the experimental data for the dependence of $\Delta U = U(z_0) - U(z)$ on the z coordinate in accordance with (3.4) and the theoretical curve corresponding to (3.5). In this figure the circles correspond to a jet with $Q = 1.89\cdot10^{-2}$ cm²/sec, $I = 0.58\cdot10^{-7}$ A, and the squares to a jet with $Q = 2.2\cdot10^{-2}$, $I = 0.63\cdot10^{-7}$.

The analogous dependence for a uniform field with an eight-centimeter interelectrode gap is shown in Fig. 3: for glycerol jets with s = 0.186 (circles), 0.181 (squares), and 0.196 (triangles).

For the jets represented in Figs. 1-3 the electric accelerations $IE/\rho Q$ varied from 30g for DBP to 900g for GL and CHL (g is the acceleration of gravity).

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